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least common multiples the group generated by the two reflections $x_1 - n$, $x_2 - n$ is the dihedral rotation group of order $2m'$.

The generalization of the case when l is arbitrary is so similar to the case considered above that it seems unnecessary to give details. The only important difference consists of the fact that m' represents the least common multiple of of m_1/d_1, m_2/d_2, where d_1 and d_2 are the highest common factors, respectively, of all such difference as $a_\alpha - a_\beta$ and $b_\alpha - b_\beta$. The numbers of the special values of n is again equal to the order of the corresponding group. It is clear that every dihedral group (including the four group) may be represented in an infinite number of ways as a subtraction group. The stress should, however, not be laid upon the fact that subtraction furnishes such interesting illustrations of this important system of groups, but rather upon the fact that these groups give a deeper and far reaching meaning to the fundamental operation of subtraction.

STANFORD UNIVERSITY, *September, 1904.*

THE SINKING-FUND OF THE UNITED STATES.

By G. B. M. ZERR.

The public debt of the United States is being paid by the sinking-fund in the following manner. During each fiscal year a sum is paid equal to one per cent. of the principal of the current debt, plus a sum equal to the interest on the part of the debt already paid at the rate of interest the debt bears. If such a sinking-fund had been operated under the same law from the beginning, how long would it require to pay the public debt, if the rate of interest the debt draws is four per cent. per annum?

A very excellent solution of the above problem is given in the *Mathematical Magazine* for September, 1904, by Theodore L. DeLand, who employs the Calculus of Finite Differences.

As the great debt of the United States will fall due in a few years, and, as its payment, then, will have to be met by a long-time loan at a different rate, which will change the present sinking-fund, we believe that a simple algebraic solution of this national problem will be interesting to the readers of the MONTHLY.

Let p=principal of the public debt at the beginning; r=.01, the rate per annum on the current principal; R=.04, the rate of interest the debt draws per annum; n=number of years required to pay the debt.

Then rp=first payment;

$p(1-r)$=unpaid part of debt after first payment;

$rp(1-r)+Rrp=pr(1-r+R)$=second payment;

$pr(1+1-r+R)=$total paid;

$p[1-r-r(1-r+R)]=$unpaid part after second payment;

$pRr(1+1-r+R)+pr[1-r-r(1-r+R)]=pr(1-r+R)^2=$thirdpayment;

$pr[1+1-r+R+(1-r+R)^2]=$total paid;

$p[1-r-r(1-r+R)-r(1-r+R)^2]=$unpaid part after third payment;

$pr[R+R(1-r+R)+R(1-r+R)^2]+pr[1-r-r(1-r+R)-r(1-r+R)^2]$
$=pr(1-r+R)^3=$fourth payment.

Similarly, $pr(1-r+R)^{n-1}=n$th payment. Hence

$$pr+pr(1-r+R)+pr(1-r+R)^2+pr(1-r+R)^3+\dots\dots+pr(1-r+R)^{n-1}=p.$$

$$\therefore pr[1+(1-r+R)+(1-r+R)^2+(1-r+R)^3+\dots\dots+(1-r+R)^{n-1}]=p.$$

$$\therefore r[(1-r+R)^n-1]/(R-r)=1.$$

$$\therefore (1-r+R)^n=R/r, \quad n=\frac{\log(R/r)}{\log(1-r+R)}.$$

$$n=\frac{\log 4}{\log(1.03)}=\frac{.6020599913}{.0128372247}=46.8995 \text{ years}=46 \text{ years, 10 months,}$$

24 days.

A PROPERTY OF THE GROUP $G_2{}^{2n}$ ALL OF WHOSE OPERATORS EXCEPT IDENTITY ARE OF PERIOD 2.

By L. E. DICKSON.

1. G is a commutative group since $ab=(ab)^{-1}=b^{-1}a^{-1}=ba$. As a concrete form of G we may take the group of the linear substitutions which multiply each of the $2n$ variables by ± 1.

It is always possible to separate the operators other than I of $G_2{}^{2n}$ into 2^n+1 sets each of 2^n-1 operators such that those of any set together with I form a group of order 2^n, and such that no two sets have a common operator. We consider the number N_n and character of all such separations into sets. Evidently $N_1=3$. We show that $N_2=56$, $N_3=2^{12}.3.5.31$.

2. Let first* $n=2$. The first set (a, b, ab) may be chosen in $\frac{1}{6}.15.14=35$ ways. The second set (A, B, AB) may then be chosen in $\frac{1}{6}.12.8$ ways, since A may be any operator except I, a, b, ab, and B any operator except these four and their products† by A. Then AB differs from I, a, b, ab, A, B. Indeed, a rectangular table of the operators of G_{16} is given by

*Cf. Ex. 2, p. 60, Burnside's *Theory of Groups*; errata, p. xvi.

†If, for example, $B=bA$, then $AB=b$ would occur in the first set.